EXPRESSION OF ALL SI UNITS BY ONE PARAMETER WITH ACCEPTABLE UNCERTAINTIES

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ABSTRACT

Finding out how many parameters are necessary to explain and describe complex and various phenomena of nature has been a challenge in modern physics. This paper introduces a new formal system of units, which maintain compatibility with SI units, to express all seven SI base units by dimensionless numbers with acceptable uncertainties and to establish the number one as the fundamental parameter of everything. All seven SI base units are converted successfully into the unified dimensionless numerical values via normalization of s, c, h, k, e/m_e, N_A, and b by unity (1). In the proposed system of units, even the unlike-dimensioned physical quantities can be convertible and hence added, subtracted, or compared to one another. It is very simple and easy to analyze and validate physical equations by substituting every unit with the corresponding number. Furthermore, it is expected to find new relationships among unlike-dimensioned physical quantities, which is extremely difficult or even impossible in SI units.

Keywords: Zero Zone system of units, Dimensionless numerical number, Qunit, SI, Fine structure constant, Rydberg constant, Uncertainty

1 INTRODUCTION

The International System of Units (SI), which is the modern form of the metric system, is universally adopted today in international trade as well as in the fields of science and technology (BIPM, 2006). The seven base quantities used in the SI and the symbols used to denote them are length (m), mass (kg), time(s), electric current (A), thermodynamic temperature (K), amount of substance (mol), and luminous intensity (cd). Units for all other quantities other than the aforementioned are mainly derived as products of powers of base units, and these units are called derived units. Some examples of quantities and the corresponding derived units are force ($N=kg{\cdot}m/s^2$), energy ($J=N{\cdot}m$), and power ($W=J/s$).

Historically, metric units were not defined in terms of universal physical constants, nor were they defined in such a manner that some chosen set of physical constants would have exact numerical values of unity. Instead, the metric system was defined from the meridian and rotation of the Earth, which in turn defined the magnitude of important physical constants.

After the French Revolution at the end of the eighteenth century, new metric units of meter and kilogram were introduced. In 1875, the system of units based on meter, kilogram, and second, namely, the MKS system was established. At this time, Stoney (1826-1911) from England was thinking about a system of units based on constants that were observed in nature rather than human standards of convenience such as the standard mass of a kilogram or the length of a meter bar. The reason behind his idea was that the existing units were anthropocentric in principle, and Stoney wanted to establish a unit system that did not depend on where we are located in the universe beyond the Earth. He proposed a system of units in 1883 based on the constants of physics, assuming they are the same everywhere in universe (Stoney, 1883). The constants that Stoney adopted were the speed of light(c), the universal gravitation constant (G), and the basic electron charge (e). It is notable that he considered the speed of light to be a constant well before Einstein's similar proposal. Stoney showed that those constants could be combined so that a unit of mass, a unit of length, and a unit of time could be derived from them.

In 1899, Planck (1858-1947) proposed a unit system that was independent of special bodies or substances by adopting natural units of mass, length, and time constructed from the most fundamental constants of nature such as the gravitation constant *G*, the speed of light *c*, and the constant of action *h*, which is now called the Planck constant (Planck, 1899). The Planck constant *h* indicates the smallest amount of energy required for action, otherwise known as a 'quantum'. Note that the Planck constant divided by 2π is called 'the reduced Planck constant' and is known as 'the Dirac constant'. In addition, by adopting Boltzmann's constant *k*, natural temperature was defined. When these four constants are properly combined, Planck's measurement units with the dimensions of mass, length, time, and temperature are derived. These are called Planck mass, Planck length, Planck time, and Planck temperature, respectively. Currently, Planck units of measurement are defined exclusively in terms of the five fundamental constants, including the Coulomb force constant, in such a way that all of these constants take on the numerical value of unity (1)(Barrow, 2004).

These units are also known as natural units because the origin of their definition does not come from any human construct, and hence they eliminate anthropocentric arbitrariness from the system of units. As a result, invariant scaling of nature becomes possible by using natural units.

By setting the numerical values of the five fundamental constants to unity, Planck units simplify many of the algebraic expressions in physics by removing conversion factors. The simplification by natural units makes them quite common in quantum gravity and high energy physics research. Another advantage of using Planck units is that if all physical quantities were expressed in those terms, the quantities would be dimensionless values. In other words, universal quantities with dimensions are normalized by the choice of natural units. Note that the five fundamental constants are all related to dynamic phenomena observed in nature.

Despite the fact that natural units including Planck units may be useful in a number of fields and more natural in a sense that they are derived from the fundamental constants, they cannot replace all SI units and cannot be applied to some areas due to the unacceptably large uncertainty of those units.

According to the values of the fundamental physical constants recommended by CODATA in 2006 (throughout this paper, we refer to the 2006 CODATA recommended values of the fundamental physics constants as CODATA 2006 (Mohr & Taylor, 2008)), the relative standard uncertainties of the Planck mass, Planck temperature, Planck length, and Planck time are about 5.0×10^{-5}, which is very large compared with the relative standard uncertainty of the same kind of fundamental physical quantities appearing in CODATA 2006 values (Mohr & Taylor, 2008). The large uncertainty in Planck units is mainly due to the large uncertainty of the Newtonian constant of gravitation, *G*, of 1.0×10^{-4}**.** As a consequence, the Planck system of units is practically considered inappropriate for deployment to all scientific areas due to its large uncertainties except for special scientific fields in which the benefits from adopting natural units are clear.

In the existing system of natural units, five physical constants were selected and set to unity, thereby allowing the conversion of five out of seven base SI units into dimensionless numbers. However, this approach has the inherent limitation in that the uncertainties are relatively high, and two basic units, the mol and cd, are not included. As a result, the existing system of natural units does not provide a methodology to convert all physical quantities into dimensionless numbers within the experimentally defined range of uncertainty while maintaining the compatibility with SI units.

In this paper, a new mathematical tool is proposed by which all seven SI base units and all physical quantities are converted into their corresponding dimensionless numeric values with the estimated relative standard uncertainties to be within an acceptable range for being deployed in all scientific areas for the first time.

In our proposed system of units, seven physical quantities are selected and set to one (1) in order to convert all seven base SI units into dimensionless numbers without any contradiction. Among the seven base SI units, the unit of time s is independently defined to be the dimensionless number 1. The remaining base SI units are calculated, depending on the selected physical constants, to produce dimensionless numbers that are proportional to s, the unit of time. In this process, physical constants with exact values including the speed of light and those with relatively lower uncertainties such as the Rydberg and fine structure constants are adopted in the derivation process for certain units in order to minimize the relative uncertainties of the converted dimensionless numbers.

Because all SI units and hence all physical quantities are expressed by their own corresponding numerical values, there are no dimensions in our proposed system of units. This implies that even the unlike-dimensioned physical quantities can be added, subtracted, or compared to one another**.** It is likely to be most attractive and useful to derive new relationships among the unlike-dimensioned physical quantities, which may be extremely difficult or impossible to do using the existing SI units. In addition, some new physical laws may be easily validated by applying the dimensionless numerical values of units.

2 ZERO ZONE SYSTEM OF UNITS

2.1 Nondimensionalization

Nondimensionalization is usually adopted for the partial or full removal of units from a mathematical equation by a suitable substitution of variables, which can simplify and parameterize problems where dimensional units are involved. The natural system of units is a typical example of nondimensionalization. Natural units are physical units of measurement defined in such a way that some chosen physical constants are normalized to have a numerical value of unity. Natural units are intended to simplify particular algebraic expressions of physical laws or to normalize some chosen physical constants that are properties of free space or universal elementary particles that may reasonably be believed to be constant. In many cases of the natural system of units including Planck units and Stoney units, the five physical quantities length, mass, time, temperature, and electric charge are adopted and defined as the quantities of base units in the natural system. Physical constants subject to normalization are selected from candidate physical constants such as the speed of light in a vacuum(c), the gravitational constant (G), Dirac's constant or reduced Planck's constant ($h/2\pi$), the Coulomb force constant ($1/4\pi\varepsilon_0$), elementary charge(e), electron mass(m_e), proton mass(m_p), and Boltzmann's constant(k).

This paper proposes a simple and generalized method of nondimensionalization by substituting all units in any physical quantity with their corresponding dimensionless values. This approach is well applied to nondimensionalization of all physical equations rather than to certain specific cases.

We define that a dimensioned physical quantity(x) is composed of a nondimensional numeric variable (x^*) and a unit (x_u), as in Equation (1).

$$x = x^* x_u \qquad (1)$$

If time duration is measured in seconds (s), e.g., t = 10 s, x is t, x_u is s, and x^* is equal to the nondimensional quantity of 10. If the unit x_u is given by its corresponding unique numeric value, the physical quantity x can easily be converted into a dimensionless numerical value by substituting the unit with its own unique value.

The proposed method of nondimensionalization is achieved by setting the dimensionless values of all seven SI base units in such a way that certain universal physical constants as well as the time duration of one second are properly selected and normalized to unity. The dimensionless values of all other derived units can also be derived from those of SI base units. Consequently, all dimensioned physical quantities can be converted into numerical values simply by substituting each unit with its numeric values.

A new term 'qunit' is coined to describe dimensionless numerical values of dimensioned physical quantities. 'Qunit value' is also used for the converted dimensionless numerical value of a physical quantity through normalization of setting certain physical quantities to unity. 'Qunit' stands for 'physical quantity embedding units in numbers'.

We define Q[x] as an operator of a physical quantity x that converts it into the qunit value by substituting all of the units in it with their respective nondimensional numerical values. For example, the qunit value of a physical quantity x as given in Equation (1) can be obtained simply, as shown in Equation (2).

$$Q[x] = Q[x^* x_u] = x^* Q[x_u] \qquad (2)$$

where x^* is the numerical coefficient of x, and $Q[x_u]$ is the nondimensional numerical value of x_u

2.2 Candidate physical constants for nondimensionalization

In order to convert all seven SI base units into dimensionless values, it is necessary to determine which physical constants should be selected to be defined as a dimensionless numeric value of unity. In addition, the uncertainty of those dimensionless values should be within compatible range, compared to that of well-known physical constants.

It is worth noting that any one of the seven SI base units can be selected to be defined by a desired numerical value independently with the other six converted to their qunit values depending on the other physical quantities selected for normalization with no conflict. Here the SI base unit of time, second (s), is selected and converted into its dimensionless numerical value as the reference for all other units.

- If we consider a physical quantity of the time duration of one second, t_1 = 1s, and normalize it by unity, we can get the qunit value of s from the relationship $Q[t_1]$ = Q[s] =1. It is understood that the duration of time of 1s, 20s, or 300s can simply be converted into the corresponding dimensionless values of 1, 20, or 300, respectively.

- The fine-structure constant, $\alpha = e^2 / 4\pi\varepsilon_0\hbar c$ = 1/137.035 999 679(94), has pure nondimensional numeric value no matter what system of units is applied. Because α is a pure dimensionless value, we can select any combination of three out of four constants comprising α, namely c, $h/2\pi$, e, and $4\pi\varepsilon_0$, for normalization by the numerical value unity but not all four constants at the same time due to over determination.

- Among the four constants above, we select the speed of light in a vacuum c and normalize it by $Q[c]$ = 1, from which the qunit value of the SI base unit of length, Q[m], can be obtained. We also select the Planck constant h for normalization by $Q[h]$ = 1 to obtain the qunit value of SI base unit of mass, Q[kg].

- For converting the SI base unit of electric current, ampere(A), into its qunit value Q[A], the ratio of elementary charge to mass electron e/m_e is selected to be defined by the numerical value of unity, that is, $Q[e/m_e]$ = 1. From this, we can get the dimensionless values of ampere Q[A] and that of kilogram Q[kg] with a specific relationship between them. The Boltzmann constant k, which can be viewed as expressing the definition of unit thermodynamic temperature, is also selected for normalization by $Q[k]$ = 1, from which the qunit value of Kelvin Q[K] can be derived.

- In order to convert the SI base unit of amount of substance, mol, into the corresponding qunit value, the Avogadro constant N_A is selected for normalization by $Q[N_A]$ = 1.

- The SI base unit of luminous intensity cd can also be converted into its qunit value by selecting a proper physical constant for normalization. The definition of cd describes how a light source emits one candela, such that the candela is the luminous intensity in a given direction of emitted monochromatic radiation of frequency 540×10^{12} hertz with a radiant intensity in that direction of 1/683 watts per steradian(sr). Then, we consider the spectral luminous efficacy B for monochromatic radiation of frequency of 540 x 10^{12} hertz as exactly 683 lumens per watt, B = 683 lm/W = 683 cd sr/W. If we select the physical constant $b=B$/sr = 638 cd/W for normalization by $Q[b]$ = 1, the converted qunit value of cd Q[cd] can be obtained.

In summary, the proposed system of units is established by selection of six physical constants in addition to one SI base unit for normalization by the nondimensional value of unity as shown in Equation (3).

$$Q[s] = Q[c] = Q[h] = Q[e/m_e] = Q[k] = Q[N_A] = Q[b] = 1 \qquad (3)$$

2.3 Derivation of qunit values for seven SI base units

As explained above, the seven postulates given in Equation (3) are applied for conversion of seven SI base units into dimensionless qunit values. First of all, the qunit value of the SI base unit of time Q[s] equals 1 simply by definition in Equation (3). It is an exact value with zero uncertainty.

$$Q[s] = 1 \quad (4)$$

The qunit value of the SI base unit of length Q[m] can easily be obtained from the defined speed of light (Equation (5)) by setting Q[s] = Q[c] = 1 as shown in Equation (3).

$$Q[c] = 299\ 792\ 458\ Q[m]/Q[s] = 1 \quad (5)$$

The result, shown in Equation (6), is also an exact value.

$$Q[m] = 1/299\ 792\ 458 \quad (6)$$

The qunit value of the SI base unit of mass Q[kg] can be obtained from the normalization of the Planck constant, h = 6.626 068 96(33) x 10^{-34} Js, by Q[h] =1 as given Equation (7). By substituting Q[s] = 1 and Q[m] as obtained above in Equation (6), the qunit value for kilogram Q[kg] is calculated as shown in Equation (8), and its relative standard uncertainty is calculated to be identical to that of the Planck constant, i.e., 5.0x10^{-8}.

$$Q[h] = 6.626\ 068\ 96(33) \times 10^{-34}\ Q[J]Q[s] = 6.\ 626\ 068\ 96(33) \times 10^{-34}\ Q[m]^2\ Q[kg]Q[s]^{-1} = 1 \quad (7)$$

$$Q[kg] = 1.356\ 392\ 733(68) \times 10^{50} \quad (8)$$

In order to obtain the dimensionless value of the SI base unit of electric current Q[A], several steps are needed. First, the dimensionless value of the electron mass Q[m_e] can be obtained from the Rydberg constant $R_\infty = \alpha^2 m_e c/2h$ = 10 973 731.568 527(73) m^{-1}, with the relative standard uncertainty of 6.6 x 10^{-12}. Its converted form in qunit is given in Equation (9).

$$Q[R_\infty] = \alpha^2 Q[m_e]Q[c]/2Q[h] = 10\ 973\ 731.568\ 527(73)\ Q[m]^{-1} \quad (9)$$

By adopting Q[c] = Q[h] = 1 as in Equation (3), the dimensionless value of Q[m] in Equation (6) and the fine structure constant $\alpha = 7.297\ 352\ 5376(50)x10^{-3}$, the qunit value of the electron mass Q[m_e] can be calculated in Equation (9). As per the postulate of Q[e/m_e] = Q[e]/Q[m_e] = 1, the dimensionless value of the elementary charge Q[e] takes the same value as that of electron mass Q[m_e] and is given by Equation (10).

$$Q[e] = Q[m_e] = 1.235\ 589\ 9746(17) \times 10^{20} \quad (10)$$

It is worth noting that the relative standard uncertainty of the dimensionless value of m_e can be calculated by substituting Q[c] = Q[h] = 1, i.e., an exact value for Equation (9), and depends only on the relative standard uncertainty of the Rydberg constant, 6.6 x 10^{-12},and that of the fine structure constant, 6.8 x 10^{-10}. It is, therefore, calculated to be 1.4 x 10^{-9}, which is better than 2.5 x 10^{-8} and 5.0 x 10^{-8}, the relative standard uncertainty of e and m_e , respectively, announced by CODATA in 2006. (Mohr & Taylor, 2008)

The elementary charge e is given by e = 1.602 176 487(40) x 10^{-19} C from which the dimensionless value of Coulomb Q[C] can be obtained directly by substituting Q[e] into the converted equation in qunit, Q[e] = 1.602 176 487(40) x 10^{-19} Q[C]. Note that, because A = C/s by definition and its converted form in qunit is given by Q[A] = Q[C]/Q[s] with Q[s] = 1, the dimensionless value for the SI base unit of electric current Q[A] is the same as that of Q[C] and is given in Equation (11).

$$Q[A] = Q[C] = 7.711\ 946\ 75(23) \times 10^{38} \quad (11)$$

The relative standard uncertainty of Q[C] can be calculated by using the calculated relative standard uncertainty of the dimensionless value of Q[e], 1.4 x 10^{-9}, as well as the given relative standard uncertainty of the constant coefficient of e , 2.5 x 10^{-8}. As a result, the relative standard uncertainty of Q[C] is calculated to be 2.9x10^{-8}

As for the SI base unit of thermodynamic temperature K, it can be expressed by the Boltzmann constant k and the SI unit of energy Joule(J) in the converted form in qunit as shown in Equation (12).

$$Q[k]Q[K] = 1.380\ 6504 \times 10^{-23}\ Q[J] = 1.380\ 6504 \times 10^{-23}\ Q[m]^2\ Q[kg]Q[s]^{-2} \qquad (12)$$

By applying Q[k] = Q[s] = 1 and the qunit values of Q[m] and Q[kg] in Equation (12), the qunit value of K is obtained in Equation (13). The relative uncertainty is calculated to be 1.7 x 10^{-6}.

$$Q[K] = 2.083\ 6644(36) \times 10^{10} \qquad (13)$$

The SI base unit for the amount of substance mol is defined as the amount of substance that contains as many elementary entities as atoms in 0.012 kilogram of carbon 12. This definition of mole also determines the value of the Avogadro constant N_A, which relates the value of entities to the amount of substance in any sample. Note that N_A mol = 6.022 141 79(30) x 10^{23} in the SI unit, and its converted form in qunit is Q[N_A]Q[mol] = 6.022 141 79(30) x 10^{23} because Q[N_A] =1 by definition. Q[mol] can be obtained simply, as shown in Equation (14), and its relative uncertainty is 5.8x10^{-8}, which is the same as was published in CODATA 2006 (Mohr & Taylor, 2008).

$$Q[mol] = 6.022\ 141\ 79(30) \times 10^{23} \qquad (14)$$

From the definition of the SI base unit of luminous intensity candela(cd), the spectral luminous efficacy(B) for the monochromatic radiation of frequency of 540 x 10^{12} hertz can be given by B = 683 (cd)(sr)/W where sr is steradian and W is Watt. If we take a sphere of unit radius, in effect, a segment of one steradian covers one quarter of the surface area of the sphere, sr equals π.

By replacing B with $b = B$/sr and W with kgm^{2}s^{-3} in SI units, the above equation can be expressed in the converted qunit form as is shown in Equation (15). Note that this unit depends on three SI base units: kg, m, and s even though it is one of the seven SI base units.

$$Q[b] = 683\ Q[cd]\ Q[m]^{-2}\ Q[kg]^{-1}\ Q[s]^{3} \qquad (15)$$

By substituting in Equation (15) Q[b] = Q[s] = 1 and the qunit value of Q[m] and Q[kg] given in Equation (6) and Equation (8) respectively, the dimensionless value of Q[cd] is obtained as shown in Equation (16). Here, the relative uncertainty is calculated to be 5.0x10^{-8}.

$$Q[cd] = 2.209\ 649\ 27(11) \times 10^{30} \qquad (16)$$

Table 1 shows the results of conversion of the seven base SI units into nondimensional numerical values. Based on this set of numeric values for the seven SI base units, it is possible to derive corresponding numeric values for all the other derived units and physical quantities.

Table 1. Summary of zero zone dimensionless values for the seven SI base units

Quantity	Unit	Dimensionless value in qunit	Calculated relative standard uncertainty*
Time	s	Q[s] = 1	Exact value
Length	m	Q[m] = 1/299 792 458	Exact value
Mass	kg	Q[kg] = 1.356 392 733(68)x10^{50}	5 x10^{-8}
Electric current	A	Q[A] = 7.711 946 75(23)x10^{38}	2.9 x10^{-8}
Thermodynamic Temperature	K	Q[K] = 2.083 664 4(36)x10^{10}	1.7 x10^{-6}
Amount of substance	mol	Q[mol] = 6.022 141 79(30)x10^{23}	5.0 x10^{-8}
Luminous intensity	cd	Q[cd] = 2.209 649 27(11)x10^{30}	5.0 x10^{-8}

* Based on uncertainty given in Mohr & Taylor (2008) the 2006 CODATA adjustment of the Fundamental Physics Constants.

Table 2 shows physical quantities with dimensionless qunit value '1' and their equivalent physical quantities in SI units. As qunit numeric values do not reveal any information about the units, it is necessary to convert them into equivalent SI physical quantities by using the conversion ratio in Table 2 to identify their physical implications. By adopting numbers in Table 1, we can establish a new numeric unit system that unifies the established SI unit system and natural unit system. This new numeric unit system is called the Zero Zone unit system.

Table 2. Zero zone dimensionless value of '1' and its equivalent physical quantity in SI unit

Quantity	Value in qunit	Equivalent physical quantity in SI unit
Zero Zone Time	1	1 s
Zero Zone Length	1	$2.997\,924\,58 \times 10^{8}$ m
Zero Zone Mass	1	$7.372\,495\,99[37] \times 10^{-51}$ kg
Zero Zone Electric Current	1	$1.269\,689\,452[38] \times 10^{-39}$ A
Zero Zone Thermodynamic Temperature	1	$4.799\,2374[82] \times 10^{-11}$ K
Zero Zone Amount of Substance	1	$1.660\,538\,782[83] \times 10^{-24}$ mol
Zero Zone Luminous Intensity	1	$4.525\,605\,10[23] \times 10^{-31}$ cd

2.4 Compatibility with SI units

One of the most unique characteristics of the Zero Zone system of units is that it allows conversion of all physical quantities as well as SI units into their dimensionless numerical values. because it is compatible with the International System of Units, even though the dimensionless value is substituted for its corresponding physical quantity or unit in any physical equation, it would not be contrary to any laws of nature that are consistent with SI units within the acceptable uncertainty. To ensure integrity here, the zero zone system of units proposed in this paper is investigated and validated for its consistency with existing SI units by comparing the standard uncertainties of the dimensionless values derived herein with those of the CODATA 2006 set of recommended fundamental physics constants (Mohr & Taylor, 2008) for some fundamental constants as well as the conversion factors between eight energy equivalent units.

According to CODATA 2006, the ratio of elementary charge to electron mass quotient e/m_e in SI units is

$$e\, m_e^{-1} = 1.785\,820\,150(44) \times 10^{11}\ \text{C kg}^{-1} \tag{16}$$

From Equation (16), we get

$$e\, m_e^{-1}\ \text{C}^{-1}\ \text{kg} = 1.785\,820\,150(44) \times 10^{11} \tag{17}$$

Using the relationship $Q[e\, m_e^{-1}\ \text{kg}\ \text{C}^{-1}] = Q[e]Q[m_e]^{-1}Q[\text{kg}]Q[\text{C}]^{-1}$, the left side of Equation (17) can be converted into a dimensionless value by adopting $Q[e] = Q[m_e]$ in Equation (10), Q[kg] in Table 1, and Q[C] in Equation (11), and we can get

$$Q[e\, m_e^{-1}\ \text{C}^{-1}\ \text{kg}] = 1.785\,820\,149 \times 10^{11} \tag{18}$$

Comparing the calculated result shown in Equation (18) with that given by CODATA 2006 values shown in Equation (17), the difference is acceptable within the permissible uncertainty range. A similar process could be applied to other physical constants such as the von Klitzing, Josephson, and Faraday constants, and the results are shown in Table 3.

Table 3. Comparison of CODATA 2006 recommended values with those calculated by using converted dimensionless values in zero zone system of units.

Related Quantity	items for comparison	From 2006 CODATA recommended values	Calculated by using the proposed qunit values
elementary charge to mass quotient	$e\, m_e^{-1}\, C^{-1}$ kg	1.785 820 150(44) x 10^{11}	1.785 820 149 x 10^{11}
Von Klitzing constant	$h\, e^{-2}\, \Omega^{-1}$	2.581 280 7557(18) x 10^4	2.581 280 7544 x 10^4
Josephson constant	$2\, e\, h^{-1}$ Hz^{-1} V	4.835 978 91(12) x 10^{14}	4.835 978 91 x 10^{14}
Faraday constant	$N_A\, e$ C^{-1} mol	9.648 533 99(24) x 10^4	9.648 533 97 x 10^4

As another example showing the uniformity of the proposed system of units, 64 conversion factors between the eight energy units J, kg, m^{-1}, Hz, K, eV, atomic mass unit(u), and Hartree energy(E_h) recommended by CODATA in 2006 are compared with those calculated by using the dimensionless numerical values proposed in this paper. Note that the derived units of these eight energy equivalent units can be calculated by using the seven SI base units in Table 1.

The 64 conversion factors calculated directly by using qunit values of eight energy units proposed in this paper are shown in Table 4. The conversion factor in each box represents the qunit value of the unit in its row divided by the qunit value of the unit in its column. For example, the value for the kg row and J column represents the conversion factor Q[kg]/Q[J] = 8.987 551 787… x 10^{16} .

It is understood that the number in parentheses is the numerical value of standard uncertainty referring to the corresponding last digits of each value. A conversion factor represented with no uncertainty, e.g., Q[m^{-1}]/Q[H_z], means exact value.

Table 4. Conversion factors calculated directly by using qunit values 8 units

unit	J	Kg	m^{-1}	Hz
J	1	1.112 650 056… x 10^{-17}	5.034 117 47(31) x 10^{24}	1.509 190 451(94) x 10^{33}
kg	8.987 551 787… x 10^{16}	1	4.524 439 15(16) x 10^{41}	1.356 392 733(68) x 10^{50}
m^{-1}	1.986 445 500(123) x 10^{-25}	2.210 218 70(13) x 10^{-42}	1	2.997 924 58 x 10^{8}
Hz	6.626 068 96(41) x 10^{-34}	7.372 496 00(43) x 10^{-51}	3.335 640 951… x 10^{-09}	1
K	1.380 6504(24) x 10^{-23}	1.536 1807(27) x 10^{-40}	6.950 356(12) x 10^{1}	2.083 6644(36) x 10^{10}
eV	1.602 176 487(154) x 10^{-19}	1.782 661 758(164) x 10^{-36}	8.065 544 65(27) x 10^{5}	2.417 989 454(82) x 10^{14}
u	1.492 417 830(95) x 10^{-10}	1.660 538 782(99) x 10^{-27}	7.523 006 671(14) x 10^{14}	2.252 342 7369(40) x 10^{23}
E_h	4.359 743 94(27) x 10^{-18}	4.850 869 34(28) x 10^{-35}	2.194 463 313(14) x 10^{7}	6.579 683 920 722 (43) x 10^{15}
J	7.242 963(13) x 10^{22}	6.241 509 65(60) x 10^{18}	6.700 536 41(43) x 10^{9}	2.293 712 69(14) x 10^{17}
Kg	6.509 651(11) x 10^{39}	5.609 589 12(52) x 10^{35}	6.022 141 79(36) x 10^{26}	2.061 486 16(12) x 10^{34}
m^{-1}	1.438 7752(24) x 10^{-2}	1.239 842 875(42) x 10^{-6}	1.331 025 0394(24) x 10^{-15}	4.556 335 252 760 (30) x 10^{-8}
Hz	4.799 2373(81) x 10^{-11}	4.135 667 33(14) x 10^{-15}	4.439 821 6294(80) x 10^{-24}	1.519 829 846 006 (10) x 10^{-16}
K	1	8.617 343(15) x 10^{-5}	9.251 098(16) x 10^{-14}	3.166 8153(54) x 10^{-6}

eV	1.160 4505(20) X 10^{4}	1	1.073 544 188(38) X 10^{-9}	3.674 932 540(125) X 10^{-2}
u	1.080 9527(18) X 10^{13}	9.314 940 28(33) X 10^{8}	1	3.423 177 7150(62) X 10^{7}
E_h	3.157 7465(54) X 10^{5}	2.721 138 386(93) X 10^{1}	2.921 262 2986(53) X 10^{-8}	1

As an example, the conversion factor for Q[kg]/Q[J] is obtained by the simplified Equation (19).

$$Q[kg]/Q[J] = Q[kg]/Q[kgm^2s^{-2}] = Q[m]^{-2}\,Q[s]^2 \tag{19}$$

In this case, Q[kg] in the numerator and denominator cancel each other out, Q[s] is an exact value of unity as shown in Equation (4), and Q[m] also has an exact value (Equation (6)). As a result, this conversion factor appears to have an exact value. However, another example of a conversion factor for Q[kg]/Q[m^{-1}] has a standard uncertainty of 0.000 000 16 x 10^{41} as shown in Table 4. In this case, even though Q[m^{-1}] or $Q[m]^{-1}$ has an exact value, Q[kg] has a relative standard uncertainty in the range of 10^{-8} order, the same as that of the Planck constant, which results in the calculated uncertainty of the conversion factor.

Note that all 64 conversion factors among the eight energy equivalent units presented in CODATA 2006 were calculated from well-known equations such as $E=mc^2=hc/\lambda=h\nu=kT$, based on the CODATA 2006 adjustment of the values of the constants. These conversion factors are obtained from the tautological relationships among units and physical constants rather than the qualitative or quantitative conceptual analysis among base units based on attributes of countable materials. For example, at the center of such a simple tautology is the well-known energy-mass equivalence. In contrast, however, the values for conversion factors given in Table 4 were calculated directly from the qunit values of two related units.

Consistency and uniformity of all 64 conversion factors obtained from the proposed Zero Zone system of units can be validated within the permissible uncertainty range compared with those in CODATA 2006.

2.5 Convertibility between unlike-dimensioned physical quantities

Dimensional analysis in its primitive form may be used to check the dimensional homogeneity of physical equations such that both sides of any equation must have the same dimensions or units. In a physically meaningful expression, only quantities of the same dimension or like-dimensioned quantities can be added, subtracted, or compared to one another. For example, the mass of a cat and the length of a dog cannot be meaningfully added, and "3 kg > 2 m" is not a meaningful expression.

In the Zero Zone system of units in which the selected physical constants are set to unity as shown in Equation (3), all seven SI base units are converted into dimensionless numerical values as shown in Table 1, which can be used to convert all physical quantities as well as derived units into the corresponding qunit values. It is worthwhile noting that physical equations such as $c = 1$ should not be taken literally because the velocity of light c has a dimension of ms^{-1}, but Q[c] is dimensionless and may be literally equal to unity.

Because there are no dimensions at all in the proposed system of units, any one of the SI base units may be converted into another with the corresponding conversion factor. Actually, those qunit values of SI base units in Table 1 can use the conversion factors among themselves. For example, we can easily get conversion factors such as Q[m]/Q[s] = 1/299792458, Q[kg]/Q[s] = 1.356392733 x10^{50}, Q[kg]/Q[m] = 4.066363114 x 10^{58} and others.

There is no doubt that unlike-dimensioned physical quantities in SI units can not be substituted in physical equations for each other. For example, the unit J can not be substituted directly by the unit kg in physical equations in SI because the conversion factor J/kg can not be determined and is not literally equal to 8.987 551 787… x 10^{16}, even though it is known that the energy of E = 8.987 551 787… x 10^{16} J is equivalent to the mass of m = 1 kg under Einstein's mass-energy equivalence, $E = mc^2$. However, if the Zero Zone system of units is applied to this case, $E = mc^2$ can be converted into the nondimensionalized form of $Q[E] = Q[m]Q[c]^2$. Because Q[c] = 1, Q[E] = 8.987 551 787… x 10^{16}. Q[J] is equal, rather than equivalent, to Q[m] = Q[kg] for a given mass of m = 1 kg, that is 8.987 551

787… x 10^{16} Q[J] = Q[kg]. Because we can get the conversion factor between the qunit values of J and kg to be Q[kg]/Q[J] = 8.987 551 787… x 10^{16} or Q[J]/Q[kg] = 1.112 650 056…x 10^{-17}, Q[J] can be substituted with Q[kg] with the above mentioned conversion factor and vice versa in all physical equations in the proposed system of units.

It is worthwhile noting that any one unit in the physical equations in the Zero Zone system of units can be substituted directly for another with the corresponding conversion factor between them having no mathematical contradiction at all. With this convertibility among units with no exceptions, a paradigm in science is shifted such that the unlike-dimensioned physical quantities can be added, subtracted, or compared to one another. It would be very attractive and helpful to find some new physical equation that may be extremely difficult or impossible to discover using SI units.

3 CONCLUSION

The so-called 'Zero Zone system of units' has been proposed where all seven SI base units are converted successfully into qunit values, i.e., unified dimensionless numerical values, via normalization of s, c, h, k, e/m_e, N_A , and b to unity. Because all other units can be derived from the seven SI base units, all physical quantities can be expressed explicitly by one parameter, that is, the pure number unity, with no dimensions at all.

Because there are no dimensional barriers among qunit values of all units, the qunit values of certain units in algebraic equations can be substituted with those of other units with the appropriate conversion factors without any mathematical contradiction. Even unlike-dimensioned units can be added, subtracted, or compared with each other by using their converted qunit values. It is easier to analyze the relationships among certain physical quantities and even to validate complex scientific equations immediately and precisely by using the qunit values of units. Besides, it is possible to predict innovative physical equations, which have unlike-dimensioned quantities on both sides of the equation.

The so-called democracy of units is realized in the newly proposed system of units based on dimensionless numeric values, given that the system does not choose a particular quantity as the fundamental physical quantity, and there is no significant difference between basic units and derived units.

In the Zero Zone system of units, the dimensionless value of the SI base unit of time s is defined independently as the exact value of unity and that of the SI base unit of length m is derived also as an exact value in relation to the velocity of light. Even though numerical values of all other base SI units are not exact values and have some uncertainties depending on the selected physical constants, they are acceptable compared with the experimentally well-established physical constants given by CODATA 2006. This implies that the system of units proposed in this paper satisfies the most fundamental requirements that are applicable to all fields of science while retaining compatibility with the established SI units.

This system of units can be validated for its consistency and compatibility with SI units by comparing the numeric coefficients of fundamental constants and 64 conversion factors among the eight energy units as given by CODATA 2006 with those calculated directly by substituting the concerned units with their converted qunit values. All the calculated values are within the boundaries of the permissible uncertainties of the experimental data given by CODATA, which satisfies one of the basic requirements of being generally applicable to physics. For improving the uncertainty of the converted numeric values of units or constants, physical constants with relatively low uncertainties such as the Rydberg and fine structure constants are used for the conversion process, in addition to physical constants with exact values. If these two constants are defined to have exact values, the uncertainty level of current adjusted physical constants could be lowered even further. Given that this is also related to the redefinition of kg and other SI base units, more serious discussion may be necessary in this regard (Mills, Mohr, Quinn, Taylor, & Williams, 2006).

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