STUDY AND APPLICATION OF GREY ENTROPY WEIGHT DECISION MAKING IN RISK MANAGEMENT

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ABSTRACT

In traditional risk evaluation, the weight of a risk index is given in advance, so it lacks objectivity. Using weights and properties generated by entropy concepts, including the idea of information entropy, the comprehensive weight, which can be combined with entropy weight, is calculated. A grey evaluation model of a project risk evaluation index based on comprehensive entropy weight is built. Further, we present empirical research on a real project, which indicates that this approach calculates easily, gives weight scientifically, and provides evaluation accurately.

Keywords: Grey entropy weight, Decision Making, Risk management

1 INTRODUCTION

Risk is a kind of measurement that measures the probability that the project objectives can not be achieved within the limit of the specified cost, progress, and technology. It means both the probability that specific targets can not be achieved and the consequences of this kind of failure (Song, Ran, &Li, 2003). Project risks include those technologies based on, planning, security, costs, and progress. Because of the complexity of these risks, procedures, methods, and techniques for risk management tend to be complex and run through the whole process of the project. Risk assessment is a very important part of project risk management. It studies the development process and the key risk areas and then identifies and records the related risks. Once the information above is obtained, we are able to rate these risk incidents by project criteria, in other words, to make an overall assessment. Risk assessment is a complex issue, because a project itself has great uncertainty and often lacks historical data and comparison with similar objects. Many factors can not be quantitatively evaluated objectively but can only be qualitative assessed. Delphi methods are often used, leading to poor objectivity and inaccurate risk assessment index weights. Risk factors for the entire project tend to be in a "some are known, and some are unknown" state, meaning grey. In the 1990s, the concept of grey-associated entropy was discovered by Zhang Qishan. Some scholars have conducted in-depth research of it in theory and applied it to various areas (Zhang, Guo, & Deng, 1996; Zhang, 2002; Luo & Liu, 2004; Li, 2005; Guo, 2005; Xie & Zhong, 2002). In this paper, we use the concept of information entropy and the decision-making method of the grey system; the grey entropy weight method for risk assessment is proposed, and a grey entropy weight model for risk assessment, which has a broad adaptability, is built (Cai, Wang, & Li, 2003).

2 THE ENTROPIES AND ENTROPY WEIGHTS OF RISK INDICATORS

The concept of entropy originated in thermodynamics. It describes the irreversible phenomenon of the movement of particles, such as molecules, atoms, or ions. Shannon later introduced it into information theory, and now it is widely used in engineering technology, socio-economics, and other fields. According to the basic principles of information theory, information is a measure of the degree of system order, while entropy is a measure of the degree of disorder. They have equal absolute value but opposite signs (Qiu, 2002; Mao, 2004). In this paper, an entropy method is used in the assessment and sorting of risk indicators, and the information contained in properties of indicators is fully used. For an index x_j, its information entropy E_j is as follows.

$$E_j = -k\sum_{i=1}^{n}(p_{ij}\ln p_{ij}) \qquad (1)$$

where $k = 1/\ln n$ and p is the probability mass function.

The assumption is made that when $p_{ij} = 0$, $p_{ij} \ln p_{ij} = 0$. For some index property *j*, if different evaluation factors are very close in this property value, according to (1), we can know that the larger the entropy of *j*, the less obvious the role of *j*. If different evaluation factors are the same for this property value, the entropy value will reach the maximum value of 1, which means the indicator j is of no importance in the comparison of plans. Therefore, the larger the difference among these indicator property values, the more information that is provided, and the more important the indicator is.

Definition 1 The entropy weight of the indicator j . $\omega_j = (1-E_j) \Big/ (m - \sum_{j=1}^{m} E_j)$ (2)

Obviously, ω_j meets $0 \le \omega_j \le 1$ and $\sum_{i=1}^{m}\omega_j = 1$.

According to (2), the smaller the indicator entropy, the larger the variation of the indicator value is, the more information is provided, and the bigger the role it plays in a comprehensive evaluation, and the larger its weight is. On the contrary, the larger the indicator entropy, the smaller the variation of the indicator value is, and the less information is provided, consequently the smaller the role it plays in the comprehensive evaluation, and the smaller its weight is. In extreme situations, if the entropy of one indicator is the maximum, $E_{\max} = 1$, it means that for each object, the values of the indicator will be same, so the indicator is no use for the evaluation and can be removed completely.

The comprehensive entropy of the indicator

Weight is the measure of the importance of goals (indicators), which is method of measuring the importance of goals. This concept of weight includes and reflects the following three factors: (1) the importance policy makers place on the goal; (2) the variation of the goal property values; and (3) the reliability of property values (Yue & Zhang, 2004).

Entropy measures the amount of useful information in available data. Evaluation of entropy indicators and entropy weights show the amount of useful information and extent of differences in property values. If policy-makers have given an indicator *j* a priori weight $w_j^{'}$ (or given by industry experts based on experience) to show the extent importance they lay on the goals, we can combine the priori weight with the entropy weight to get the comprehensive posterior weight, as follows.

$$w_j = \frac{w_j^{'}\omega_j}{\sum_{j=1}^{m} w_j^{'}\omega_j}$$

(3)

So formula (3) can calculate the comprehensive weight of an indicator, and formula (2) shows how to obtain ω_j. It contains and reflects the three factors that should be reflected by weight in a comprehensive and reasonable way. According to the indicator property value table $D^{'} = (z_{ij})_{n\times m}$ and formula (3), we obtain the comprehensive weight of each indicator w_j . The comprehensive entropy weight matrix for each indicator is as follows.

$$W = (w_1, w_2 \cdots w_m) \qquad (4)$$

Supposing there is a multi-attribute decision-making problem MA(m, n), $Y_i = (y_{i1,} y_{i2,\dots} y_m)$, $X = \{x_1, x_2, \dots x_n\}$ means the property values of x_i. When the objective function is $y_{ij} = f_j(x_i)$, $i = 1,\cdots,n; j = 1,\cdots m.$, the property value of each project can be listed as attribute value tables or attribute matrices $D = (y_{ij})_{n\times m}$. To give the actual value of the property in evaluating a project, we need the normalization of property value table above. Then we get:

$$D^{'} \ : \ D^{'} = (p_{ij})_{n\times m},\ p_{ij} \in [0,1]$$

$$X = (x_{ij})_{n\times m,}\ x_{ij} \ge 0, (i = 1,2,\cdots,n;\ j = 1,2,\cdots,m). \qquad p_{ij} = x_{ij} \Big/ \sum_{i=1}^{n} x_{ij}$$

3 THE GREY EVALUATION BASED ON THE GREY ENTROPY

Definition 2 Suppose the number of cluster objects is n, the cluster indicators m, and different grey classes s. According to the observation $x_{ij}(i=1,2,\cdots,n;\ j=1,2,\cdots,m)$, about the indicator j for the object I (i=1,2,…,n), we put object i into the grey class $k(k\in\{1,2,\cdots,s\}$. This is known as the grey cluster (Zhang, 2002).
Definition 3 According to the indicator j value of n objects, these objects are divided into grey clusters s which are called indicator subclasses j. The whitenization weight function of indicator j subclass k is recorded as $f_j^k(\cdot)$

Definition 4 The whitenization weight function f_j^k of indicator j subclass k shown in Figure 1 is the typical whitenization weight function. $x_j^k(1), x_j^k(2), x_j^k(3), x_j^k(4)$ is called the turning points of $f_j^k(\cdot)$. The typical whitenization weight function is recorded as $f_j^k[x_j^k(1), x_j^k(2), x_j^k(3), x_j^k(4)]$.

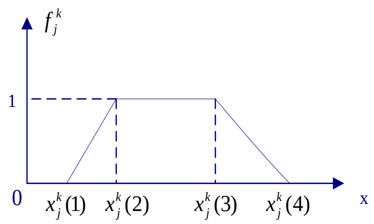

Figure 1.The typical whitenization weight function

Definition 5 (1) If the whitenization weight function $f_j^k(\cdot)$ doesn't have the first and second turning points $x_j^k(1), x_j^k(2)$, as is shown in Figure 2., then $f_j^k(\cdot)$ is called lower measure whitenization weight function and recorded as $f_j^k[-,-,x_j^k(3), x_j^k(4)]$.

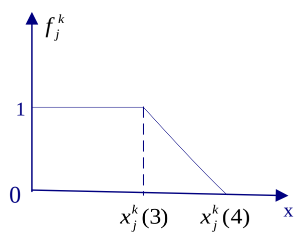

Figure 2. The lower measure whitenization weight function

(2) If the whitenization weight function $f_j^k(\cdot)$ number 2 and number 3 turning points overlap, as shown in Figure 3, then $f_j^k(\cdot)$ is called moderate measure whitenization weight function and recorded as
$f_j^k[x_j^k(1), x_j^k(2), -, x_j^k(4)]$.

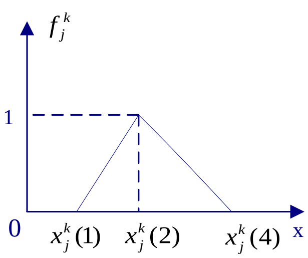

Figure 3. The moderate measure whitenization weight function

(3) If the whitenization weight function $f_j^k(\cdot)$ does not have the third and fourth turning points $x_j^k(3), x_j^k(4)$, as is shown in Figure 4, then $f_j^k(\cdot)$ is called the upper measure whitenization weight function and recorded as $f_j^k[x_j^k(1), x_j^k(2), -, -]$ (Liu & Dang, 2004).

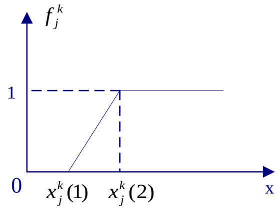

Figure 4. The upper measure whitenization weight function

The steps for the evaluation of the Grey Entropy weight

(1)The grey classes, the whitenization weight function, and the grey evaluation coefficients are calculated. For indicator $V(v_1, v_2, \cdots v_j \cdots v_m)$, the evaluation coefficient $X_{ij}^{(k)}$ of the grey class k is calculated as follows (Mi, Liu, Dang, & Fang, 2006).

$$X_j^k = \sum_{i=1}^{n} f_j^k(x_{ij})(i = 1, \cdots, n;\ j = 1, \cdots, m;\ k = 1, \cdots, s)$$

For indicator V_j, the total evaluation coefficient for each grey class X_j meets the criteria that . $X_j = \sum_{k=1}^{s} x_j^k (k = 1, \cdots, s)$

(2) The grey evaluation weight vector and matrix are calculated. For the grey evaluation weight of the evaluation indicator V_j in grey class k for all evaluators, the evaluation matrix of indicator for each grey class is $r_j = (r_j^1, r_j^2, \cdots, r_j^s)$.

(3) The grey evaluation matrix of all indicators is evaluated.

The matrix is evaluated by the following:

$$R = \begin{bmatrix} r_1^1, r_1^2, \cdots, r_1^s \\ r_2^1, r_2^2, \cdots, r_2^s \\ \cdots\cdots\cdots\cdots \\ r_m^1, r_m^2, \cdots, r_m^s \end{bmatrix} \tag{5}$$

(4) The risk evaluation index V is comprehensively evaluated.

The index is calculated by:

$$r_j^k = \frac{X_j^k}{X_j} \tag{6}$$

$$G_m = W \bullet R = (w_1, w_2, \cdots, w_m) \bullet \begin{bmatrix} r_1^1, r_1^2, \cdots, r_1^s \\ r_2^1, r_2^2, \cdots, r_2^s \\ \cdots\cdots\cdots\cdots \\ r_m^1, r_m^2, \cdots, r_m^s \end{bmatrix} \tag{7}$$

$$= (g_1, g_2, \cdots, g_m)$$

$$G = \sum_{i=1}^{m} g_m \tag{8}$$

(5) The risk evaluation index V is evaluated by the grey method. The index is evaluated by:

$$\sigma = W \bullet R = (\sigma^1, \sigma^2, \cdots \sigma^s) \tag{9}$$

$$\sigma^k = \max\{\sigma^1, \sigma^2, \cdots \sigma^s\} = \sigma^{k*} \tag{10}$$

4 CASE STUDY

The hierarchical structure of risk assessment and the structure of risk indicators are shown in Figure 5. Project risk indicators on the first levels cover technical, financial, planning, security, and market risks.

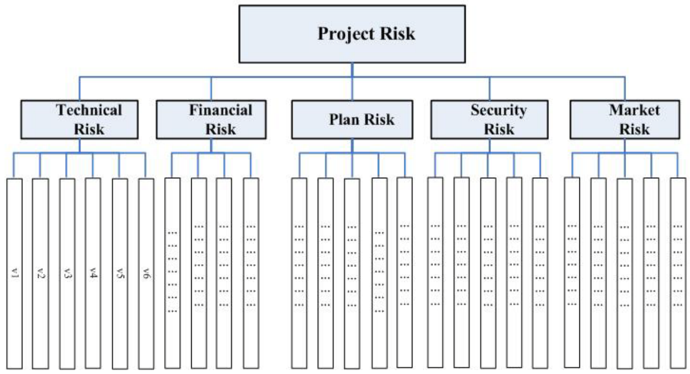

Figure 5. The architecture of risk indicators in one project

Taking technical risks as an example, the factors impacting technical risks are uncertainty of the outcome of research and development v_1 , the possibility that the technology cannot meet the requirements v_2 , the possibility that production conditions cannot meet the requirements v_3 , the uncertainty of the life of the technology v_4, the uncertainty of of technological change happening v_5 , and the uncertainty involved in the circumstances of the application v_6 .

The comprehensive evaluation of technical risk entropy weight

Suppose a technical risk observation matrix is as follows.

$$D = \begin{bmatrix} 7 & 7 & 9 & 7 & 7 & 3 \\ 7 & 5 & 7 & 7 & 7 & 3 \\ 7 & 5 & 7 & 7 & 7 & 3 \\ 7 & 5 & 7 & 7 & 5 & 3 \\ 7 & 5 & 5 & 5 & 5 & 3 \\ 5 & 3 & 5 & 5 & 5 & 3 \\ 5 & 3 & 3 & 5 & 5 & 1 \\ 5 & 3 & 3 & 5 & 5 & 1 \end{bmatrix}$$

According to the normalization matrix above, using formulas (1), (2), and (3), we can calculate the entropy and its weight for each indicator for technical risks. In Table 1 we add prior weights of importance that policy makes place on the goals in column 4 and then use the formula (4) to calculate the comprehensive entropy.

Table1. Entropy, entropy weight and comprehensive entropy weight of risk index

Risk Index	Entropy	Entropy Weight ω_j	Experience Weight ω_j	Comprehensive Entropy Weight w_j
V1	3.7037	0.1686	0.2	0.2028
V2	3.6487	0.1661	0.25	0.2497
V3	3.6144	0.1646	0.35	0.3463
V4	3.7009	0.1685	0.1	0.1013
V5	3.7011	0.1685	0.05	0.0507
V6	3.596	0.1637	0.05	0.0492

W = [0.2028 0.2497 0.3463 0.1013 0.0507 0.0492]

The risk indicators have five grey classes: A, B, C, D, and E. The whitenization weight functions given by the experts, taking into account that every indicator is normalized and the whitenization weight functions are only different in levels, meaning that they are the same in other property values, which simplifies the problem, are as follows:

$$f_j^1[0,9,-,-]\ ,\ f_j^2[-,-,7,14]\ ,\ f_j^3[-,-,5,10]\ ,\ f_j^4[-,-,3,6]\ ,\ f_j^5[-,-,1,2]$$

Applying steps (1) and (2), we get

$$r_1 = (0.2614, 0.3764, 0.3011, 0.0612, 0.0000)$$

Similarly, we get r_2, r_3, r_4, r_5, and r_6. Applying step (3), we get

$$R = \begin{bmatrix} 0.2614 & 0.3764 & 0.3011 & 0.0612 & 0.0000 \\ 0.1674 & 0.3347 & 0.3180 & 0.1799 & 0.000 \\ 0.2922 & 0.3267 & 0.2287 & 0.1525 & 0.0000 \\ 0.2532 & 0.3797 & 0.3038 & 0.0633 & 0.0000 \\ 0.1594 & 0.3909 & 0.3322 & 0.0814 & 0.0000 \\ 0.0787 & 0.2835 & 0.2835 & 0.2835 & 0.0709 \end{bmatrix}$$

Step (4) gives the comprehensive evaluation of the technical risk as:

$$G_1 = W \bullet R \ = (0.2336, 0.3453, 0.2812, 0.1346, 0.0035)$$

$$G_v = \sum_{i=1}^{5} g_m = 0.9983$$

The Grey evaluation of the technical risk

$$\sigma = W \bullet R = (\sigma^1, \sigma^2, \cdots \sigma^s)\ ,\ \sigma^k = \max\{\sigma^1, \sigma^2, \cdots \sigma^s\} = \sigma^{k*}$$

$$\sigma = (0.2354, 0.3453, 0.2812, 0.1346, 0.0035)$$

$$\sigma^k = 0.3453$$

We determine that k = 2 and conclude the technical risks have a higher value, that is, B-class. Similarly we can do a comprehensive evaluation of other risks, as shown in Table 2.

From the calculation above, we know that among the project risk evaluation indicators on level one, the plan risks are the highest both for the impact and on the risk levels. Because the plan risks result from improper decisions made by people, they bring to the project both the greatest risk and risk on the highest level.

Table 2. Results of the evaluation of risk indices

Risk Index	Technical Risk	Funding Risk	Plan Risk	Security Risk	Market Risk
Evaluation results	0.9983	0.7897	1.2253	0.4951	0.6930
Indexes ranking	2	3	1	5	4
Indexes category	B	C	A	D	C

The impact of the project risk levels rank as follows.

Plan risks > Technical risks > Financial risks > Market risks > Security risks

The levels of risk rank as follows.

Plan risk > Technical risk > (Market risk = Financial risk) > Security risk

If we only consider the project itself and no other factors, the risk levels caused by capital (financial) and market are the same.

5 CONCLUSIONS

In this paper, by using entropy weights, grey theory and adopting a more objective method to determine the weight (called comprehensive entropy weight method) and studying risk assessment in risk management, the grey assessment method based on the entropy weight is put forward. It provides solutions to the sort of risk indicators on the same level and the definition of risk levels. The empirical study shows that this method is simple and accurate and has high operability and can be used in risk assessment of many industries and sectors.

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