RESEARCH OF GPS ANTI-JAMMING BASED ON CIRCULAR ANTENNA ARRAY

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ABSTRACT

In this paper, a new model for suppressing jammers to GPS receivers is proposed. In the model, circular antenna arrays combining minimum norm (min-norm) and linearly constrained minimum variance (LCMV) algorithms have been used for signal anti-jamming. Six GPS signals' and two jammers' original incident direction were assumed respectively. The simulation was performed with a variation of the power of the two jammers and the element number of the circular antenna array. The simulation result indicates that by utilizing this new signal suppression model, nulls depths assigned to the jammer reach -238dBW when the number of element of circular antenna array is assumed to be 30. It also indicates that the stronger power of the jammer, the deeper nulls depths can be assigned with this new signal processing structure.

Key words: Circular Antenna Array, Far field radiation, Minimum Norm Algorithm,Linearly Constrained Minimum Variance (LCMV), Global Positioning System (GPS)

1 INTRODUCTION

The GPS receiver is the best device in the field of navigation to give a very accurate user position. Therefore, it is used in many civilian and military applications. Interference from radar systems and other devices affects civilian use and intentionally used jammers affect military use; accordingly increasing the protection against intentional and unintentional interference is required. The received GPS signal is about -160dBW, i.e., it is below the receiver thermal noise power by about 20-30 dBW (Kaplan, 1996). The adaptive antenna (Figure 1) is suitable to cancel these types of jammers. It utilizes some cancellation techniques for determining the jammer direction based on its power such as power inversion (PI) (Compton, 1971; Schwegman, et al., 1972; Zahm, 1973; Compton, 1979) multiple signal classification (MUSIC) (LU, et al., 2001), and Minimum Norm Algorithms (Kumaresan, et al., 1983). When dealing with GPS anti-jam, the main purpose of the adaptive antenna is to reduce the jamming signals up to a level where the spread spectrum mechanism can extract a useful signal. The LCMV algorithm is one of the most efficient algorithms for canceling a jammer from an unknown direction. The new signal anti-jamming model structure is introduced in section 2. In section 3, the main algorithms used in this paper, such as minimum norm and LCMV, are introduced. Section 4 gives the simulation results. Finally the conclusion is given in section 5.

2 MODEL STRUCTURE FOR GPS ANTI-JAMMING

There are different types of antenna arrays, such as M-elements linear array, M-elements circular array, rectangular array, etc. The total electric field radiated by an antenna array can be given as:

$$\mathbf{E}_{\text{total}} = \mathbf{E}_{\text{Single element at the origin}} \times \text{Array Factor}$$

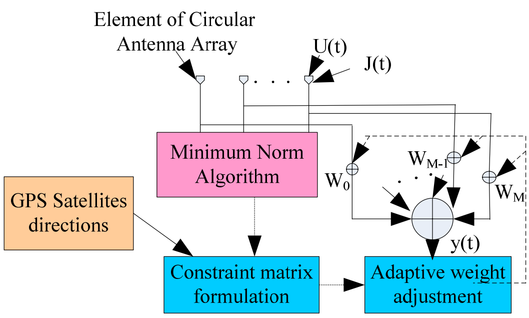

Figure 1. Model structure: Min-norm and LCMV algorithms combining with circular antenna array

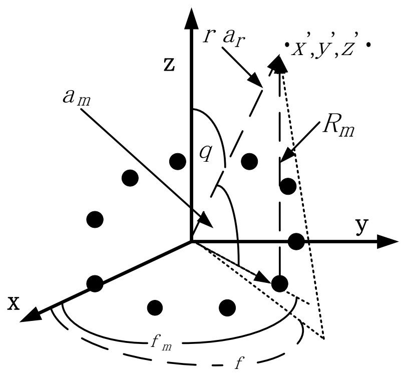

Figure 2. Circular array configuration

Consider M-infinitesimal dipoles are arranged equally spaced along a circle of radius b as shown in Figure 2, where b equals the wavelength of GPS L1. α_m is the angle between a_r and a_ρ

$$\cos\alpha_m = a_\rho . a_r = (a_x \cos\phi_m + a_y \sin\phi_m)\cdot(a_x \sin\theta\cos\phi + a_y \sin\theta\sin\phi + a_z \cos\theta)$$
$$= \sin\theta\cos(\phi - \phi_m)$$

For far field radiation, the amplitude variation $R_m \cong r$ .The electric field of the circular array is given as follows,

$$E(r,\theta,\phi) = \sum_{m=1}^{M} a_m \cdot \frac{e^{-jkR_m}}{R_m}$$
$$= \frac{e^{-jkr}}{r}\sum_{m=1}^{M} a_m e^{jkb\sin\theta\cos(\phi-\phi_m)}$$

where $a_m = C_m e^{jg_{cm}}$. C_m, g_{cm} are the amplitude and phase excitation of the m^{th} element. So, we can get,

$$E(r,\theta,\phi) = \frac{e^{-jkr}}{r}\sum_{m=1}^{M} C_m e^{j(kb\sin\theta\cos(\phi-\phi_m)+g_{cm})}$$

then

$$A(\theta,\phi) = \sum_{m=1}^{N} C_m e^{j(kb\sin\theta\cos(\phi-\phi_m)+g_{cm})}$$

3 MINIMUM NORM AND LCMV ALGORITHM

3.1 Minimum norm algorithm

Consider the received signal at M-elements uniformly spaced circular array is linear combination of all the far field incident signals and noise. Thus,

$$\mathbf{X}=\mathbf{V}\mathbf{u}+\mathbf{N} \tag{1}$$

$$y = \mathbf{w}^H \mathbf{x} \tag{2}$$

$\mathbf{X}(n) = (x_1(n)\ x_2(n)\cdots x_M(n))^T$ a $M \times 1$ vector represents the antenna array received signal.

$\mathbf{V} = \left(\alpha\begin{pmatrix}\theta_1 \\ \varphi_1\end{pmatrix} \; \alpha\begin{pmatrix}\theta_2 \\ \varphi_2\end{pmatrix} \cdots \alpha\begin{pmatrix}\theta_L \\ \varphi_L\end{pmatrix} \right)$ a matrix contains the steering vectors associated to the incident signal.

$\mathbf{u} = (u_1(n) \;\; u_2(n) \cdots u_L(n))^T$ a vector represents the incident signals amplitudes.

$\mathbf{w} = (w_0 \;\; w_1 \cdots w_{M-1})^T$ an $M \times 1$ complex vector represents the array weight vector.

y is the output of the array antenna given by the weighted sum of the array antenna received signal.

$\mathbf{N} = (n_1(n) \; n_2(n) \cdots n_M(n))^T$ an $M \times 1$ vector consists of an independent Gaussian noise of variance σ^2 and includes channel noise, receiver noise and antenna elements noise.

Because the GPS signal is very weak; it lies in the noise subspace. Therefore, the received signal is mainly determined by the noise and the jammers. It is assumed that the jammer signals are independent of each other and independent of thermal noise. If the vector **w** is linear combination of the $M - L$ noise subspace eigenvectors, then it has the property that:

$$V^H w = 0 \tag{3}$$

Then there is a polynomial (Kumaresan, et al., 1983; Kumareasan, 1983)

$$\xi(z) = \sum_{k=1}^{M} w_{k-1} z^{-(k-1)}$$

that has L of its zeros at $\exp(j\psi_k)$, k=1, 2,…L. The $M - L$ extraneous zeros of $x(z)$ are uniformly distributed inside the unit circle in sectors where the L signals zeros are absent, if G which is given by (4) is minimum,

$$G = \sum_{k=0}^{M-1} \left| w_k \right|^2, \quad w_0 = 1 \tag{4}$$

Let

$$\xi(z) = D_1(z) D_2(z)$$
$$D_1(z) = \sum_{k=1}^{L} h_k z^{-(k-1)}, \quad h_1 = 1$$
$$D_2(z) = \sum_{k=1}^{M-L} b_k z^{-(k-1)}, \quad b_1 = 1 \tag{5}$$

Minimizing G is the same as minimizing

$$\int_{-\pi}^{\pi} \left| \xi(e^{j\psi_k}) \right|^2 d\psi \tag{6}$$

The autocorrelation matrix can be written in terms of its eigenvalues and eigenvectors as follows

$$R = QLQ^H \tag{7}$$

Where in,

$$L = \mathrm{diag}(\lambda_1 \; \lambda_2 \, \mathrm{L} \; \lambda_M), \; Q = (q_1 \; q_2 \, \mathrm{L} \; q_M)$$

λ_i represents the i^{th} eigenvalue of R , where $i = 1, 2 \cdots M$, q^i is the i^{th} eigenvector associated with the i^{th} eigenvalue of R, where $i = 1, 2 \cdots M$.

The incident jammer signals' subspace is represented by the eigenvectors corresponding to the L largest eigenvalues. The remaining [L+1:M] eigenvectors span the noise subspace. As a result:

$$(Q_{Signal})^H w = 0, \quad (Q_{Signal\,new})^H w_{new} = -a^* \tag{8}$$

$$Q_{Signal} = (a^T / Q_{Signal\,new}) \tag{9}$$

$$\mathbf{w} = (1 \quad w_2 \cdots w_M)^T = (1 \mid \mathbf{w}_{new})^T \tag{10}$$

$\mathbf{a}^T = (q_{11} \quad q_{21} \cdots q_{L1})$ represents the first elements of each of the jammer eigenvectors. The vector **w** can be constructed as follows:

$$\mathbf{w} = \left(\frac{1}{-\mathbf{Q}_{signal_{new}} \left(\left[\mathbf{Q}_{signal_{new}} \right]^H \mathbf{Q}_{signal_{new}} \right)^{-1} \mathbf{a}^*} \right)$$

3.2 Linearly constrained minimum variance (LCMV) algorithm

The main objective of LCMV is to minimize the mean squared output $E\langle |y|^2 \rangle$

$$\min |y|^2 = \min \mathrm{w}^H \mathrm{R} \mathrm{w} \tag{11}$$

The solution to (11) is given as:

$$\mathrm{w}_o = \mathrm{R}^{-1} \mathrm{C} \left(\mathrm{C}^H \mathrm{R}^{-1} \mathrm{C} \right)^{-1} \mathrm{f} \tag{12}$$

When there are no useful or jammer signals and only uncorrelated noise, (12) can be written as:

$$\mathbf{w}_{qs} = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{f} \tag{13}$$

Where $\mathbf{f} = \left(f_1 \quad f_2 \cdots f_K \right)^T$.

For adaptively calculating the weight vector, we use Lagrange multipliers to change the constrained equation (11) to unconstrained one, then:

$$\wp = \mathbf{w}^H \mathbf{R} \mathbf{w} + \boldsymbol{\lambda}^H \left(\mathbf{C}^H \mathbf{w} - \mathbf{f} \right) + \left(\mathbf{w}^H \mathbf{C} - \mathbf{f}^H \right) \boldsymbol{\lambda} \tag{14}$$

Minimizing the output power means taking the gradient of (14) with respect to $\mathbf{w}^H$ and equating the result to zero.

$$\frac{\partial \wp}{\partial \mathbf{w}^H} = 2\mathbf{R}\mathbf{w} + 2\mathbf{C}\boldsymbol{\lambda} = \mathbf{0} \tag{15}$$

λ is a $K \times 1$ Lagrange multiplier vector. Utilizing the steepest descent technique to iteratively update the weight vector, thus:

$$\mathrm{w}(n+1) = \mathrm{w}(n) - \frac{1}{2} \mu \frac{\partial \wp}{\partial \mathbf{w}^H} \tag{16}$$

Using both (15) and $\mathbf{f} = \mathbf{C}^H \mathbf{w}(n+1)$, which is the constraint part of (11) in (16), $\boldsymbol{\lambda}$ can be obtained as:

$$\boldsymbol{\lambda} = \frac{1}{\mu}\left[\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H(\mathbf{I}-\mu\mathbf{R})\mathrm{w}(n) - \left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{f}\right] \tag{17}$$

Using (16) in (17)

$$\begin{aligned} \mathrm{w}(n+1) &= \mathbf{A}\mathrm{w}(n) - \mu\mathbf{A}\mathbf{R}\mathrm{w}(n) + \mathbf{w}_{qs} \\ &= \mathbf{A}(\mathbf{I}-\mu\mathbf{R})\mathrm{w}(n) + \mathbf{w}_{qs} \end{aligned} \tag{18}$$

Considering the instantaneous value of the autocorrelation matrix so, $\mathbf{R} = \mathbf{X}\mathbf{X}^H$, (18) can take the form:

$$\mathbf{w}(\mathrm{n}+1) = \mathbf{A}(\mathbf{w}(\mathrm{n}) - \mu\mathbf{x}(\mathrm{n})\,\mathrm{y}(\mathrm{n})) + \mathbf{w}_{qs} \tag{19}$$

where $y(\mathrm{n}) = \mathbf{w}^H(\mathrm{n})\mathbf{x}(\mathrm{n})$.

4 SIMULATIONS

Computer simulations were performed using six useful GPS signals each with power -160dBW coming from six different directions and two jammers each with different power coming from two different directions. The six useful GPS signals' incident directions were assumed to be coming from:

$$\begin{pmatrix}\theta\\ \varphi\end{pmatrix} = \left(\begin{pmatrix}0^0\\ 15^0\end{pmatrix}\begin{pmatrix}15^0\\ 15^0\end{pmatrix}\begin{pmatrix}30^0\\ 30^0\end{pmatrix}\begin{pmatrix}60^0\\ 60^0\end{pmatrix}\begin{pmatrix}20^0\\ 45^0\end{pmatrix}\begin{pmatrix}45^0\\ 90^0\end{pmatrix}\right)$$

q is the angle between the incident signal plane and z axis. f is the angle between the incident signal plane and xy plane. (q and f are shown in Fig.2)

Two jammers coming directions are

$$\begin{pmatrix}\theta\\ \phi\end{pmatrix} = \left(\begin{pmatrix}45^o\\ 180^o\end{pmatrix}\begin{pmatrix}60^o\\ 90^o\end{pmatrix}\right)$$

In all the following power pattern figures, the 0dB corresponds to direction of the useful signals, and all others levels are related to this value. The range of angle θ is within $(0^0 \sim 90^0)$ and angle ϕ is within $\left(0^0 \sim 360^0\right)$.

The requirement is to achieve un-attenuated response to the directions of the useful signals and nullify the jammer coming from fixed directions with fixed power. The antenna elements assumed to be uniform and spaced equally along a circle. The number of elements in the circular antenna array is varied. So the constraint response takes the form:

$$\mathbf{f} = \begin{pmatrix}1 & 1 & 1 & 1 & 1 & 1 & 0 & 0\end{pmatrix}^T,$$

and the constraint matrix takes the form:

$$\left(u\begin{pmatrix}0^0\\ 15^0\end{pmatrix} u\begin{pmatrix}15^0\\ 15^0\end{pmatrix} u\begin{pmatrix}30^0\\ 30^0\end{pmatrix} u\begin{pmatrix}60^0\\ 60^0\end{pmatrix} u\begin{pmatrix}20^0\\ 45^0\end{pmatrix} u\begin{pmatrix}45^0\\ 90^0\end{pmatrix} J\begin{pmatrix}45^0\\ 180^0\end{pmatrix} J\begin{pmatrix}60^0\\ 90^0\end{pmatrix}\right)$$

Case 1: Two jammers come with power -120dBW and -100dBW respectively. The corresponding signal to jammer plus noise ratio at the input for the two jammers are 0.0001and 0.000001 respectively. The power pattern levels for the jammer signals according to the number of antenna elements are summarized in Table 1.

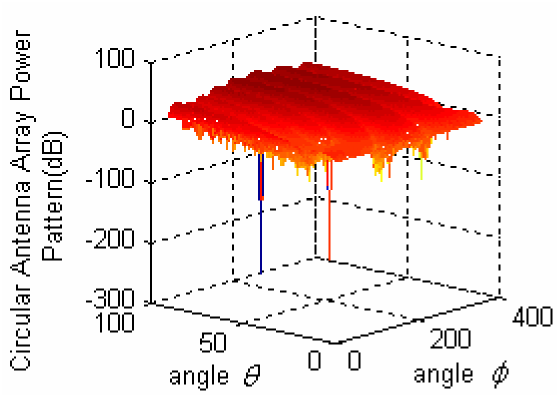

Figure 3. Antenna array power pattern in case of using 15 antenna elements with two fixed jammers utilizing minimum norm and LCMV algorithms

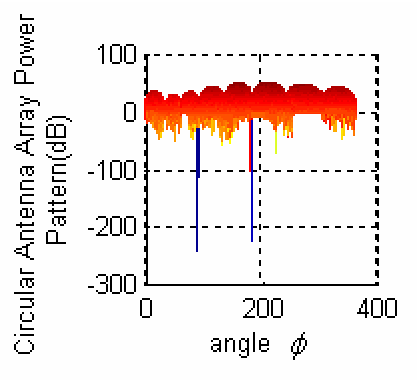

Figure 4. Angle ϕ section plane of Figure 2

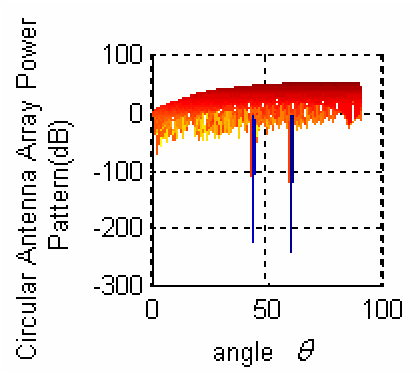

Figure 5. Angle θ section plane of Figure 2

Table 1. Antenna power pattern level in the case of varied antenna elements

The number of Antenna array elements	Angle in θ degrees	Angle in Φ degrees	Power pattern Level in dB
30	45^o 60^o	180^0 90^0	-238 -224
15	45^o 60^o	180^0 90^0	-226 -230
8	45^o 60^o	180^0 90^0	-224 -228
5	45^o 60^o	180^0 90^0	0 -220

From Table 1 we can see the difference between two jammers according to the number of antenna elements. When the number of antenna elements is 30, the weak jammer from $(45^0 \quad 180^0)$ is -227dB and the strong power jammer from $(60^0 \quad 90^0)$ is -238dB. The strong jammer is suppressed about 11dB deeper than the weak one. In this case, the nulls depth suppressed by a linear antenna array is only-119dB (Chinese Journal of Electronics, 2005). There is nothing shown in the antenna power pattern figure when the number of antenna elements is small other than the total number of jammers and useful GPS signals.

Case 2: This case is exactly like case (1), but the power of the two jammers is varied. The simulation was performed using 15 elements in a uniform circular array with elements equally spaced along a circle. The power pattern levels for the jammer signals according to the variational power are summarized in Table 2.

From Table 2, it can be seen that the circular antenna array combining with minimum norm and LCMV algorithms can highly suppresses the jammers. The strong power jammer is more deeply suppressed than the weak one. When the power of the jammers is closer to the GPS signals, however, it is very hard to find the accurate direction of the jammers.

Table 2. Antenna power pattern in case of varied power of jammers

	The power of two jammers (dBW)	Angle in θ degrees	Angle in Φ degrees	Power pattern Level in dB
1	-100	45^o	180^0	-238
	-130	60^o	90^0	-224
2	-120	45^o	180^0	-226
	-100	60^o	90^0	-230
3	-140	45^o	180^0	-224
	-100	60^o	90^0	-228
4	-160	45^o	180^0	0
	-140	60^o	90^0	-220

5 CONCLUSION

The circular antenna array combining with minimum norm and LCMV algorithms can be very efficient in suppressing the signal gaining in the direction of jammers. The modeling results show that the greater the number of antenna elements used, the better the result for suppressing the jammers gain. It also indicates that the stronger power jammer can be suppressed higher than the weak one. This is very hard to do when the power of the jammers is closer to the GPS signals.

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